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Self-affine roughness influence on the Casimir effect

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In this work we investigate the influence of self-affine roughness on the Casimir energy for plate-plate geometry. The plate roughness is characterized by the rms roughness amplitude w, the lateral correlation length ξ, and the roughness exponent H. It is shown that the latter has a prominent effect on the Casimir energy with respect to the long-wavelength roughness ratio w/ξ. Analytic results are shown for three characteristic roughness exponents, namely, $H=0$, 0.5, and 1. Moreover, the dynamic evolution of the surface roughness (e.g., by metal deposition) leads to significant contributions of the roughness influence on the Casimir energy. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1935127]

As device dimensions for microelectromechanical systems (MEMS) decrease, forces, such as the Casimir effect[1] that are normally neglected in macrosystems, have to be considered. Indeed, more than 50 years ago the eminent Dutch physicist H. Casimir discovered that two parallel conducting plates separated by a distance d attract each other with a force proportional to the surface area A or $F_{\mathrm{PP}} = -A(\pi^2\hbar c/240d^4)$.[1] This is a prediction of quantum electrodynamics, which arise from the perturbation of zero-point vacuum fluctuations of the electromagnetic field by the conducting plates. The fundamental nature of the Casimir effect and its implications, e.g., on surface forces,[2] particle physics,[3] cosmology,[4] etc., has lead to wide theoretical work.[5–9]

The first experimental attempt to measure the Casimir force was inconclusive due to large error ($\sim 100\%$).[10] Recently, it became feasible high-precision measurements ($\sim 1\%-5\%$) of the Casimir force using a torsion pendulum,[11] an atomic force microscope,[12] and a micromachined torsional device.[13] The latter also revealed the possibility for actuators based on the Casimir force.[14] Note that the van der Waals force is always attractive,[15] while the sign of the Casimir force is geometry dependent. The interpretation of the Casimir force in terms of changes in zero-point vacuum electromagnetic energy suggests that it is a function of plate geometry[16–18] that confines vacuum fluctuations. It can be repulsive, e.g., spherical shells (if a thin spherical conducting shell is cut in half, the two hemispheres will experience a mutual *repulsive* force) or cubic shells.[6,7,17] Repulsive Casimir forces are expected also when magnetic and electric properties are included.[16,19]

Although the theoretical calculation of the Casimir force has been performed for flat metal plates, in reality surfaces are never perfectly smooth on a microscopic length scale. In Ref. 20 the authors showed that the Casimir force is increased by the surface roughness. For one periodically rough plate (wavelength Λ and amplitude A_r), the Casimir energy is given by the simple analytic asymptotic expressions $E_{\mathrm{PP}_s}=E_{\mathrm{PP}}[1+3(A_r{}^2/\Lambda^2)]$ if $\Lambda \gg d$, and $E_{\mathrm{PP}_s}=E_{\mathrm{PP}}[1+2\pi(A_r{}^2/\Lambda d)]$ if $\Lambda \ll d$ with $E_{\mathrm{PP}}=-(\pi^2\hbar c/720d^3)$.[21] Nonetheless, any investigation for the roughness influence for the case of random self-affine rough plates is still missing, and it will be the topic of the present paper. In this case, besides of the rms roughness amplitude w and the lateral correlation length ξ, the short-wavelength roughness also plays critical role. The latter is characterized by a roughness exponent $H(0<H<1)$, which is a measure of the degree of surface irregularity.[23] Notably, this type of morphology occurs during metal film growth, which is necessary (e.g., Al films[12]) for various systems designed to probe the Casimir force.

Recently, it was shown that the influence of random surface roughness on the Casimir energy is given (ignoring roughness crosscorrelations) by[22]

$$E_{\mathrm{PP},r} \simeq E_{\mathrm{PP}} + \frac{1}{2}\left(\frac{\partial^2 E_{\mathrm{PP}}}{\partial d^2}\right)\sum_{m=1}^{2}\int \frac{d^2q}{(2\pi)^2} P_m(q)\langle|h_m(q)|^2\rangle, \tag{1}$$

with $\langle|h_m(q)|^2\rangle$ as the roughness spectra of both surfaces ($\langle h_m\rangle=0$). $P_m(q)$ is given by $P_m(q)=[G_{\mathrm{TM}}(qd/2\pi)+G_{\mathrm{TE}}(qd/2\pi)]/[G_{\mathrm{TM}}(0)+G_{\mathrm{TE}}(0)]$.[22] The functions G_{TM} and G_{TE} correspond to the contributions of TM (transverse-magnetic) and TE (transverse-electric) modes.[21] In the limit of large plate distance so that $d \gg \lambda_P$ with λ_P as the surface-plasmon wavelength [e.g., for Al $\lambda_P \approx 100$ nm [Ref. 12], we have $P_m(q) \simeq C_0 qd$ with $C_0=1/3$ and $qd>1$.[22] Upon substitution in Eq. (1) we obtain

$$E_{\mathrm{PP},r} \simeq E_{\mathrm{PP}} + \frac{1}{2}\left(\frac{\partial^2 E_{\mathrm{PP}}}{\partial d^2}\right)(C_0 d)C_{\mathrm{rou}},$$
$$C_{\mathrm{rou}} = \sum_{m=1}^{2}\int q\langle|h_m(q)|^2\rangle \frac{d^2q}{(2\pi)^2}. \tag{2}$$

Furthermore, the calculation of the roughness influence in Eq. (2) requires knowledge $\langle|h_m(q)|^2\rangle$. Indeed, a wide variety of surfaces and interfaces appearing in various physical systems (i.e., films grown under nonequilibrium conditions) posses self-affine roughness.[23] In this case, the roughness spectrum $\langle|h(q)|^2\rangle$ shows the power-law scaling $\langle|h(q)|^2\rangle$

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$\propto q^{-2-2H}$ if $q\xi \gg 1$ and $\langle |h(q)|^2\rangle \propto$ const if $q\xi \ll 1$.[23] This scaling is satisfied by the analytic model $\langle |h(q)|^2\rangle = [2\pi w^2\xi^2/(1+aq^2\xi^2)^{1+H}]$ with $a=1/2H[1-(1+aQ_c^2\xi^2)^{-H}](0<H<1)$ and $a=1/2\ln(1+aQ_c^2\xi^2)(H=0)$.[24] $Q_c=\pi/a_0$ with a_0 of the order of atomic dimensions. Note that small values of $H(\sim 0)$ characterize extremely jagged or irregular surfaces, while large values of $H(\sim 1)$ surfaces with smooth hills and valleys.[23]

Since $E_{\mathrm{PP}} = A(\pi^2\hbar c/720)d^{-3}$ Eq. (2) can be written in the form

$$E_{\mathrm{PP},r} \simeq E_{\mathrm{PP}}\left(1 + \frac{6C_0}{d}C_{\mathrm{rou}}\right),$$
$$C_{\mathrm{rou}} = \sum_{m=1}^{2}\int_{Q_d}^{Q_c} q^2 \frac{w_m{}^2\xi_m{}^2}{(1+aq^2\xi_m{}^2)^{1+H_m}}dq. \tag{3}$$

Equation (3) is similar to the roughness correction $E_{\mathrm{PP},r} \simeq E_{\mathrm{PP}}\{1+6(w^2/d^2)\}$[12,20] where, however, the effect of the roughness exponent H and the lateral correlation length ξ is absent. Analytic results for the roughness term C_{rou} are obtained for $H=0$, 0.5, and 1,

$$C_{\mathrm{rou}}|_{H=0} = \sum_{m=1}^{2} w_m{}^2\left\{\frac{1}{a_m}(Q_c-Q_d) - \frac{1}{a_m{}^{3/2}\xi_m} \times[\tan^{-1}(X_{\mathrm{mc}}) - \tan^{-1}(X_{\mathrm{md}})]\right\}, \tag{4}$$

$$C_{\mathrm{rou}}|_{H=0.5} = \sum_{m=1}^{2} w_m{}^2\left\{\frac{1}{a_m{}^{3/2}\xi_m} \times[\sinh^{-1}(X_{\mathrm{mc}}) - \sinh^{-1}(X_{\mathrm{md}})] - \frac{1}{a_m}\left[Q_cT_{\mathrm{mc}}{}^{-1/2} - Q_dT_{\mathrm{md}}{}^{-1/2}\right]\right\}, \tag{5}$$

$$C_{\mathrm{rou}}|_{H=1} = \sum_{m=1}^{2} w_m{}^2\left\{\frac{1}{a_m{}^{3/2}\xi_m}[\tan^{-1}(X_{\mathrm{mc}}) - \tan^{-1}(X_{\mathrm{md}})] - \frac{1}{2a_m}\left[Q_cT_{\mathrm{mc}}{}^{-1} - Q_dT_{\mathrm{md}}{}^{-1}\right]\right\}, \tag{6}$$

with $X_{\mathrm{mc}} = \sqrt{a_m}\xi_m Q_c$, $X_{\mathrm{md}} = \sqrt{a_m}\xi_m Q_d$, $Q_d = 2\pi/d$, $T_{\mathrm{mc}} = 1+(X_{\mathrm{mc}})^2$, and $T_{\mathrm{md}} = 1+(X_{\mathrm{md}})^2$. In general, C_{rou} will have a simple dependence on the roughness amplitude w since $\langle |h(q)|^2\rangle \propto w^2$, while any more complex dependence will arise from the parameters H and ξ.

Our calculations were performed assuming the same roughness parameters for both plates and a cutoff $c = 0.3$ nm. Figure 1 shows the calculations of C_{rou} as a function of the correlation length ξ. A fast decay of the roughness contribution occurs with increasing ξ (corresponding to smoothing at large wavelengths) and relatively large roughness $H(>0.5)$. For smaller exponents H the variation of C_{rou} with increasing ξ is clearly weaker. Figure 2 depicts directly the sensitive dependence of C_{rou} on the roughness exponent H. As the latter varies from 0 to 1, C_{rou} can vary by some orders of magnitude especially with larger correlation lengths ξ. If we compare Figs. 1 and 2 we can infer that the roughness exponent H has a more dominant effect on the roughness contribution with respect to the correlation length ξ or alternatively the roughness ratio w/ξ.

Finally, we will consider how the dynamic evolution of the plate roughness can influence the roughness contribution on the Casimir energy. Indeed, the plate surface morphology can be, for example, modified by metal deposition prior to any experiment (e.g., Al in the experiment of Ref. 12 where the Casimir force was measured by an atomic force microscope). Indeed, both roughness parameters w and ξ can evolve with deposition time τ as $w \propto \tau^{\beta}$ (β known as the growth exponent) (Ref. 23) and $\xi \propto \tau^{1/z}$ (z known as the dynamic exponent) (Ref. 23). If $z=H/\beta$ the local surface slope

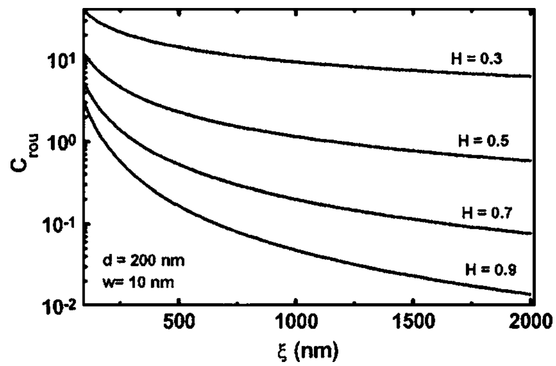


FIG. 1. Calculations of the roughness factor C_{rou} as a function of the lateral correlation length ξ for various roughness exponents H.

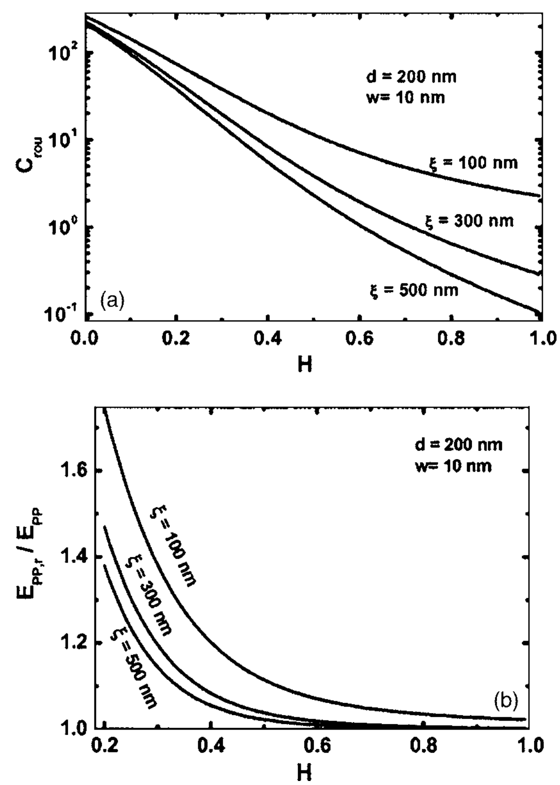
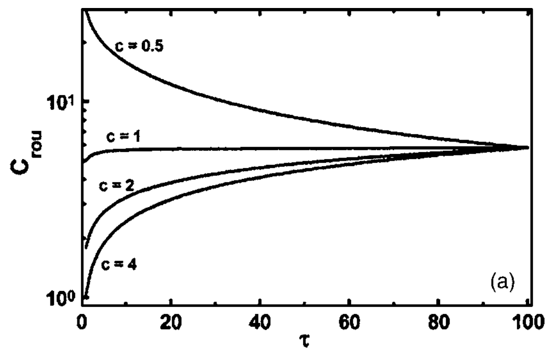


FIG. 2. (a) Calculations of the roughness factor C_{rou} as a function of the roughness exponent H for various lateral correlation length ξ (b) Contribution of the roughness exponent H on the Casimir energy ratio $E_{\mathrm{PP},r}/E_{\mathrm{PP}}$ for plate separation $d=200$ nm.



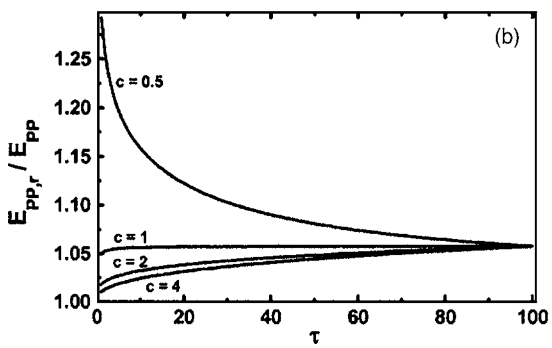


FIG. 3. (a) Calculations of the roughness factor C_{rou} as a function of the evolution time time $\tau, \beta=0.25, H=0.5$, and $z=c\beta/H$. In both cases we considered the power-law relations $w=10(\tau/100)^{\beta}$ nm and $\xi=200(\tau/100)^{1/z}$ nm. (b) Casimir energy ratio $E_{\mathrm{PP},r}/E_{\mathrm{PP}}$ (for plate separation $d=200$ nm) as a function of roughness evolution time τ from part (a).

$\rho_{\mathrm{rms}}=\sqrt{\langle|\nabla h|^2\rangle}$ is time invariant $(\partial\rho_{\mathrm{rms}}/\partial\tau=0)^{25}$ leading to weak evolution of the roughness term C_{rou} (Fig. 3). If, however, $z=cH/\beta$ with $c>1$ then surface roughening occurs (Fig. 3). Indeed, since $\rho_{\mathrm{rms}}\propto w/\xi^{H}$ (Ref. 25) we obtain the temporal evolution $\rho_{\mathrm{rms}}\propto\tau^{\beta(1-1/c)}$ which leads to surface roughening for $c>1$. On the other hand, if $c<1$ the local slope decreases at later evolution times leading to smoothening and decrement of the roughness contribution on the Casimir effect [see also Fig. 3(b)]. Note that the apparent convergence of all curves in Fig. 3 at a single point is due to the assumption at $\tau=100$ we have $w=10$ nm and $\xi=100$ nm and we present how the roughness term C_{rou} evolves for $\tau<100$. Therefore, dynamic roughness growth can also influence significantly the Casimir energy.

While the Casimir force is too small for plates that are microns apart (e.g., two plates with an area of 1 cm^2 separated by 1 μm have an attractive Casimir force $\sim 10^{-7}$ N, at distances of 10 nm it produces the equivalent of ~ 1 bar of pressure, which is rather large. For example, in MEMS the Casimir force can cause device elements to stick together,[26] or alternatively it can be used to control mechanical motion of devices.[13] Therefore, the knowledge of the roughness morphology in these systems and its influence on the Casimir effect is of potential technological importance.

In conclusion, we investigated the influence of self-affine plate roughness on the Casimir energy. The roughness exponent H is shown to have a prominent effect with respect to the long-wavelength roughness ratio w/ξ. Analytic results were derived for three characteristic roughness exponents, namely, $H=0$, 0.5, and 1. Moreover, it is shown that dynamic evolution of the plate surface roughness (e.g., by metal deposition which is typical for experiments) leads to significant contributions of the roughness influence on the Casimir effect. Indeed, plate roughness is a challenge in measuring and calculating the Casimir force since real mirrors are not perfectly smooth. Most mirrors are made by coating a substrate with a thin metal film using the sputtering technique that produces films with a roughness of ~ 30 nm (Ref. 12) or more which affect measurements of the Casimir force since it is very sensitive to small changes in distance. Thus, experiments will be need to relate further self-affine roughness characteristics and growth mode to Casimir effect.

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